EDEXCEL STATISTICS S4 (6686) – JUNE 2003 PROVISIONAL MARK SCHEME

Question number	Scheme	Marks
1.	$P(X > 2.85) = 0.05$	B1
	$P(X < \frac{1}{5.67}) = 0.01$	B1
	$\therefore P(\frac{1}{5.67} < X < 2.85) = 1 - 0.05 - 0.01$	M1
	$= 0.94$	A1
		(4 marks)
2.	$H_0: \sigma^2 = 4$; $H_1: \sigma^2 > 4$ both	B1
	$\upsilon = 19$, $X^2_{19}(0.05) = 30.144$ 30.144	B1
	$\frac{(n-1)S^2}{\sigma^2} = \frac{19 \times 6.25}{4} = 29.6875$ use of $\frac{(n-1)S^2}{\sigma^2}$	M1
	AWRT 29.7	A1
	Since $29.6875 < 30.144$ there is insufficient evidence to reject H_0.	A1 ft
	There is insufficient evidence to suggest that the standard deviation is greater than 2.	B1 ft
		(6 marks)
3. (a)	$P(X \le c_1) \le 0.05$; $P(X \le 3 \mid \lambda = 8) = 0.0424 \Rightarrow X \le 3$	M1; A1
	$P(X \ge c_2) \le 0.05$; $P(X \ge 4 \mid \lambda = 8) = 0.0342 \Rightarrow X \ge 13$	M1; A1
	$P(X \ge 13 \mid \lambda = 8) = 0.0638$	
	$\therefore$ critical region is $\{X \le 3 \cup X \ge 13\}$	A1 ft (5)
(b) (i)	$P(4 \le X \le 12 \mid \lambda = 10) = P(X \le 12) - P(X \le 3)$	M1 M1
	$= 0.7916 - 0.0103$	
	$= 0.7813$	A1
(ii)	Power $= 1 - 0.7813 = 0.2187$	B1 ft (4)
		(9 marks)

Question number	Scheme	Marks
4.	d: 7 2 –3 1 –1 –2 10 5	M1
	$\Sigma d = 19$; $\Sigma d^2 = 193$	
	$\therefore\ \bar{d} = \frac{19}{8} = 2.375$; $S_d^2 = \frac{1}{7}\left\{193 - \frac{19^2}{8}\right\} = 21.125$	B1; M1 A1
	H_0: $\mu_D = 0$; H_1: $\mu_D > 0$ both	B1
	$t = \dfrac{2.375 - 0}{\sqrt{\frac{21.125}{8}}} = 1.4615\ldots..$ AWRT 1.46	M1 A1
	$\upsilon = 7 \Rightarrow$ critical region: $t > 1.895$ 1.895	B1
	Since 1.4915… is <u>not</u> in the critical region there is insufficient evidence to reject H_0 and we conclude that there is in sufficient evidence to support the doctors' belief.	A1 ft
		(9 marks)
	Alternative:	
	Use of 2 sample t-test $\Rightarrow$ B0 B0 B0 M1 A1 M1 A1 B1 A1 i.e : 6/9 max	
	$S_p^2 = \dfrac{7\times 440.125 + 7\times 501.357}{8+8-2} = 470.74$	M1 A1
	$t = \dfrac{216.125 - 213.75}{\sqrt{470.74\left(\frac{1}{8}+\frac{1}{8}\right)}} = 0.0547$	M1 A1
	critical region: $t > 1.761$	B1
	Conclusion as above	A1 ft

Question number	Scheme	Marks
5. (a)(i)	E $(\hat{\theta}) = \theta$	B1
(ii)	E $(\hat{\theta}) = \theta$ or E $(\hat{\theta}) \to \theta$	B1
	and Var $(\hat{\theta}) \to 0$ as n $\to \infty$ where n is the sample size	B1 (3)
(b)	$\text{E}(\hat{p}_1) = p, \therefore$ Bias = 0	B1
	$\text{E}(\hat{p}_2) = \frac{5p}{6}, \therefore$ Bias $= \frac{1}{6}p$	B1 B1
	$\text{E}(\hat{p}_3) = p, \therefore$ Bias = 0	B1 (4)
(c)	$\text{Var}\left(\hat{p}_1\right) = \frac{1}{9n^2}\{npq + npq + npq\}$	M1
	$= \frac{pq}{3n}$	A1
	$\text{Var}\left(\hat{p}_2\right) = \frac{1}{36n^2}\{npq + 9npq + npq\} = \frac{11pq}{36n}$	A1
	$\text{Var}\left(\hat{p}_3\right) = \frac{1}{36n^2}\{4npq + 9npq + npq\} = \frac{7pq}{18n}$	A1 (4)
(d) (i)	$\hat{p}_1$; unbiased and smallest variance	B1 dep; B1
(ii)	$\hat{p}_2$; biased	B1 dep; B1 (4)
		(15 marks)

Question number	Scheme	Marks
6. (a)	$\bar{x} = 123.1$	B1
	$s = 5.87745….$	B1
	(NB: $\Sigma x = 1231$; $\Sigma x^2 = 151847$)	
(i)	95% confidence interval is given by	
	$123.1 \pm 2.262 \times \frac{5.87745....}{\sqrt{10}}$	M1
	2.262	B1
	i.e: (118.8958…, 127.30418…)	A1 ft
	AWRT (119, 127)	A1 A1
(ii)	95% confidence interval is given by	
	$\frac{9 \times 5.87745...^2}{19.023} < \sigma^2 < \frac{9 \times 5.87745...^2}{2.700}$ use of $\frac{(n-1)s^2}{\sigma^2}$	M1
	19.023	B1
	2.700	B1
	i.e; (16.34336…, 115.14806….)	A1ft
	AWRT (16.3, 115)	A1 A1 (13)
(b)	130 is just outside confidence interval	B1
	16 is just outside confidence interval	B1
	Thus supervisor should be concerned about the speed of the new typist	B1 (3)
		(16 marks)

Question number	Scheme	Marks
7. (a)	$S_A^2 = \frac{1}{10}\{3960540 - \frac{6600^2}{11}\} = 54.0$	B1
	$S_B^2 = \frac{1}{12}\{7410579 - \frac{9815^2}{13}\} = 21.1\dot{6}$	B1
	H_0: $\sigma_A^2 = \sigma_B^2$; H1: $\sigma_A^2 \neq \sigma_B^2$	B1
	CR: $F_{10, 12} > 2.75$	
	$S_A^2 / S_B^2 = \frac{54.0}{21.1\dot{6}} = 2.55118\ldots$	M1 A1
	Since 2.55118… is not in the critical region we can assume that the variances are equal.	B1 (6)
(b)	H_0: $\mu_B = \mu_A + 150$; H_1: $\mu_B > \mu_A + 150$ both	B1
	CR: $t_{22}(0.05) > 1.717$ 1.717	B1
	$S_p^2 = \frac{10\times 54.0 + 12\times 21.1\dot{6}}{22} = 36.0\dot{9}0\dot{9}$	M1 A1
	$t = \frac{1755 - 6001 - 150}{\sqrt{36.0909...\left(\frac{1}{11}+\frac{1}{13}\right)}} = 2.03157$	M1 A1
	AWRT 2.03	A1
	Since 2.03… is in the critical region we reject H_0 and conclude that the mean weight of cauliflowers from *B* exceeds that from *A* by at least 50g.	A1 ft (8)
(c)	Samples from normal populations	
	Equal variances Any two sensible verifications	B1 B1 (2)
	Independent samples	
		(16 marks)